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Systematics of moduli stabilization, inflationary dynamics and power spectrum

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Abstract: We study the scalar sector of type IIB superstring theory compactified on Calabi-Yau orientifolds as a place to find a mechanism of inflation in the early universe. In the large volume limit, one can stabilize the moduli in stages using perturbative method. We relate the systematics of moduli stabilization with methods to reduce the number of possible inflatons, which in turn lead to a simpler inflation analysis. Calculating the order-of-magnitude of terms in the equation of motion, we show that the methods are in fact valid. We then give the examples where these methods are used in the literature. We also show that there are effects of non-inflaton scalar fields on the scalar power spectrum. For one of the two methods, these effects can be observed with the current precision in experiments, while for the other method, the effects might never be observable.

Keywords: Flux compactifications, Cosmology of Theories beyond the SM.

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1. Introduction

The use of both RR and NS-NS fluxes to generate potentials for the moduli appearing in Calabi-Yau compactifications of Type IIB string theory [1, 2] has breathed new vigor into attempts to find inflation in the effective 4-D field theory associated with such compactifications. The generic expectation is that the potentials for the moduli fields could be flat enough to allow for a phase of slow-roll inflation for at least one, if not more, moduli. This expectation has been borne out in a number of examples [3 – 5, 21].

The idea of modular inflation from string theory has been around for some time. The earlier attempts on modular cosmology concentrated on potentials for the dilaton. However, a detailed study [7] of general properties of these potentials shows that they are either of the runaway type or too steep for inflation.

One of the difficulties in trying to find inflationary regimes for these potentials is that typically, more than one field will participate in the slow-roll phase. As an example, in [3], four fields were relevant to generating inflation. Following such a system is a complex task, and it is not unreasonable to ask whether there are ways to reduce the number of inflaton fields that need to be tracked. In this paper, we will list two ways of simplifying

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the analysis by reducing the number of possible inflatons. These methods have strong ties to the systematics of moduli stabilization in the large volume scheme that were discussed in ref. [6].

We will start by reviewing the scalar sector of type IIB superstring compactified on a large volume Calabi-Yau orientifolds. Following ref. [6], we will show that on the large volume limit, the potential for the moduli will have a scale hierarchy. We then exploit this hierarchy to approach the problem of moduli stabilization in several stages. It turns out that this hierarchy also allows us to integrate out some of the moduli from the theory and simplify the analysis for inflation.

The existence of more than one modulus in the problem can also influence the power spectrum of metric perturbations observed in the CMB. If these moduli have not yet settled into their minima during the inflationary phase, their oscillations about these minima could imprint itself into the power spectrum [8]. Given the new WMAP [9] results, we may be able to place bounds on how quickly these fields had to have reached their minima. Once we establish the hierarchical scale structure alluded to in the previous paragraph, we estimate the size of these effects. We find that in some cases, the effect could be detectable.

2. Review of the scalar sector of type IIB superstring theory

Type IIB superstring theory compactified on Calabi-Yau orientifolds M yields the following four dimensional effective theory:

$$\mathcal{L} = \int d^4x\sqrt{-g}\left(G_{\alpha\bar{\beta}}\partial_\mu\Phi^\alpha\partial^\mu\Phi^{\bar{\beta}} + V\right), \tag{2.1}$$

where α, β run over all moduli, $G_{\alpha\bar{\beta}} = \partial_\alpha\partial_{\bar{\beta}}K$ is the Kähler metric on the moduli space, and where K is the Kähler potential, including the α' corrections [10]:

$$K = -\log\left[-i\int_M \Omega\wedge\bar{\Omega}\right] - \log\left[-i(\tau-\bar{\tau})\right] - 2\log\left[\frac{\xi}{2}\left(\frac{-i(\tau-\bar{\tau})}{2}\right)^{3/2} + e^{-3\phi_0/2}\left(\mathcal{V}\right)^{2/3}\right]. \tag{2.2}$$

Here τ is the axion-dilaton field, Ω is the (3,0)-form of the Calabi-Yau, $(\mathcal{V})^{2/3}$ is the classical volume of M in units of $l_s = (2\pi)\sqrt{\alpha'}$, and $\xi = -\zeta(3)\chi(M)/(2(2\pi)^3)$. We require that $\xi > 0$, or $h^{2,1} > h^{1,1}$.

The scalar potential is given by:

$$V = e^K\left(G^{\alpha\bar{\beta}}D_\alpha W\bar{D}_{\bar{\beta}}\bar{W} - 3|W|^2\right), \tag{2.3}$$

where the superpotential W is

$$W = \int_M G_3\wedge\Omega + \sum_i A_i\, e^{ia_i\rho_i}. \tag{2.4}$$

The first term is the Gukov-Vafa-Witten term [11] and the second one is the non-perturbative part due to D3-brane instantons [12] or gaugino condensation from wrapped D7-branes (see [13] and references therein). Here $G_3 = F_3 - \tau H_3$, with F_3 and H_3 are RR and NS-NS 3-form fluxes, respectively, A_i is a one-loop determinant and $a_i = 2\pi/N$, with N is a positive integer. Also, $\rho_i \equiv b_i + i\tau_i$ is the complexified Kähler modulus consisting of the

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four-cycle modulus τ_i:

$$\tau_i = \partial_{t^i}(\mathcal{V})^{2/3} = \frac{1}{2} D_{ijk}\, t^j t^k, \tag{2.5}$$

and the axion b_i. The t^i measures the area of two-cycle, D_{ijk} are the triple intersection numbers of the divisor basis [14] and the classical volume is expressed as

$$(\mathcal{V})^{2/3} = \frac{1}{6} D_{ijk}\, t^i t^j t^k. \tag{2.6}$$

Equations (2.2) and (2.4) completely specify the theory and the problem of moduli stabilization becomes the problem of finding solutions to $\partial_\alpha V = 0$. However, visualizing the full potential and finding its minima is a difficult task as is using the full potential to look for inflationary phases. To do this, we would have to solve the following equations of motion in an FRW background

$$\ddot{\Phi}^A + 3H\dot{\Phi}^A + \Gamma^A{}_{BC}\dot{\Phi}^B\dot{\Phi}^C + 2G^{AB}V_{,B} = 0, \tag{2.7}$$

$$3H^2 = G_{AB}\dot{\Phi}^A\dot{\Phi}^B + V, \tag{2.8}$$

where $\Gamma^A{}_{BC}$ is the connection on the metric of the moduli space[1]. The slow-roll condition is $\epsilon \ll 1$ (for more details, see Appendix A), where the slow-roll parameter is given by

$$\epsilon = \frac{G_{AB}\dot{\Phi}^A\dot{\Phi}^B}{H^2}. \tag{2.9}$$

As discussed in the previous section, it is almost impossible to deal with the plethora of moduli that appear in these compactifications, at least as far as inflationary dynamics is concerned. What we need is a controlled way to "freeze" some of these into place at their minima, while leaving a subset of them free to induce an inflationary state for the requisite amount of time.

Thus, we want to somehow *consistently* decouple some fields, collectively labeled $\{\psi^A\}$, from the inflationary dynamics by putting them at the minima of the potential and let the rest of the field $\{\phi^A\}$ be the inflatons, i.e.:

$$\ddot{\phi}^A + 3H\dot{\phi}^A + \Gamma^A{}_{BC}\dot{\phi}^B\dot{\phi}^C + 2G^{AB}V_{,B} = 0, \tag{2.10}$$

$$3H^2 = G_{AB}\dot{\phi}^A\dot{\phi}^B + V. \tag{2.11}$$

In this scenario, the slow-roll condition now becomes

$$\epsilon = \frac{G_{AB}\dot{\phi}^A\dot{\phi}^B}{H^2} \ll 1. \tag{2.12}$$

The problem in doing this is that there is no reason to expect that a given choice of $\{\psi^A\}$ will work. In general, the solution $\{\psi^A{}_{\rm min}\}$ to $\partial_{\psi^A} V = 0$ will be $\{\phi^A\}$-dependent. If $\psi^A{}_{\rm min} = \psi^A{}_{\rm min}\left(\{\phi^B\}\right)$ then

$$\dot{\psi}^A_{\rm min} = \frac{\partial \psi^A{}_{\rm min}}{\partial \phi^B}\dot{\phi}^B \neq 0. \tag{2.13}$$

Thus, a careless choice of $\{\psi^A\}$ could give the false impression that the slow-roll parameter is small, i.e. inflation is occurring, when in reality ϵ might not be small.

[1]Capitalized Roman letters denote *real* scalar fields as opposed to the complex ones indicated by the Greek indices.

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Furthermore, even if $\dot{\psi}^A = 0$, there is the possibility that $\ddot{\psi}^A$ will feed on the last two terms of the equation of motion (2.7), such that on a later time $\dot{\psi}^A$ will deviate significantly from 0.

A valid choice of $\{\psi^A\}$ should not have the above problems. We will show that the systematics of moduli stabilization in the large volume scenario is strongly related to finding the valid choice of $\{\psi^A\}$.

3. Systematics of moduli stabilization in the large volume limit

Following [6], we will be working on the large volume limit, which is defined as the limit where all $\tau_i \to \infty$ except one, which we denote by τ_s, with $\tau_s \sim \ln(\mathcal{V})^{2/3}$. In this limit, the potential becomes

$$\begin{aligned} V \equiv \quad & e^K \left(G^{a\bar{b}} D_a W \bar{D}_b \bar{W} + G^{\tau\bar{\tau}} D_\tau W \bar{D}_\tau \bar{W} \right) \\ & + \left(e^K \frac{\xi}{2(\mathcal{V})^{2/3}} \left(W \bar{D}_\tau \bar{W} + \bar{W} D_\tau W \right) + V_{\alpha'} + V_{\rm np1} + V_{\rm np2} + V_{\rm uplift} \right) \\ & + \left(V_{\rm supp1} + V_{\rm supp2} + V_{\rm supp3} + V_{\rm supp4} \right), \end{aligned} \tag{3.1}$$

where

$$\begin{aligned} V_{\alpha'} &= 3\xi e^K \frac{\xi^2 + 7\xi(\mathcal{V})^{2/3} + (\mathcal{V})^{2/3^2}}{((\mathcal{V})^{2/3} - \xi)(2(\mathcal{V})^{2/3} + \xi)^2} |W|^2, \\ V_{\rm np1} &= e^K G^{\rho_s\bar{\rho}_s} a_s^2 |A_s|^2 e^{-2a_s\tau_s}, \\ V_{\rm np2} &= e^K G^{\rho_s\bar{\rho}_l} i a_s (A_s e^{i a_s \rho_s} \bar{W} \partial_{\bar{\rho}_l} K - \bar{A}_s e^{-i a_s \bar{\rho}_s} W \partial_{\rho_l} \bar{K}), \\ V_{\rm uplift} &= \frac{\epsilon_{\rm uplift}}{(\mathcal{V})^{2/3^3}}, \\ V_{\rm supp1} &= e^K\, G^{\rho_l\bar{\rho}_m} \left(a_l A_l a_m \bar{A}_m e^{i(a_l\rho_l - a_m\bar{\rho}_m)} \right) \\ V_{\rm supp2} &= e^K i\, G^{\rho_l\bar{\rho}_m} \left(a_l A_l e^{i a_l \rho_l} \bar{W} \partial_{\bar{\rho}_m} K - a_m \bar{A}_m e^{-i a_m \bar{\rho}_m} W \partial_{\rho_l} K \right) \\ V_{\rm supp3} &= e^K G^{\rho_l\bar{\rho}_s}\, 2\,{\rm Re} \left[a_l A_l a_s \bar{A}_s e^{i(a_l\rho_l - a_s\bar{\rho}_s)} \right], \\ V_{\rm supp4} &= e^K i\, G^{\rho_l\bar{\rho}_s} a_l (A_l e^{i a_l \rho_l} \bar{W} \partial_{\bar{\rho}_s} K - \bar{A}_l e^{-i a_l \bar{\rho}_l} W \partial_{\rho_s} \bar{K}). \end{aligned} \tag{3.2}$$

The indices a, b run over the complex structure moduli, and the Kähler moduli indices $l, m \neq s$. The uplift potential $V_{\rm uplift} \geq 0$ is obtained by adding anti-D3-branes [15] or by using the supersymmetric D-terms [16]. For simplicity, let us assume that all a_i's are of $\mathcal{O}(1)$, which corresponds to the gauge rank $N \lessapprox 10$ (we will discuss the case for smaller a_i in the last section). The first term is positive definite and of $\mathcal{O}((\mathcal{V})^{2/3^{-2}})$, the second is of $\mathcal{O}((\mathcal{V})^{2/3^{-3}})$, and the third is $\mathcal{O}((\mathcal{V})^{2/3^{-2/3}} e^{-(\mathcal{V})^{2/3}})$.

The hierarchy between terms in the potential allows us to approach the problem of moduli stabilization in three stages perturbatively using the inverse volume $(\mathcal{V})^{2/3^{-1}}$ as our expansion parameter. First, we will stabilize the axion-dilaton and complex structure moduli $\{\Phi_I\}$ by minimizing the leading term in the potential. Next, by including the second

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term, we will stabilize b_s, τ_s, and $(\mathcal{V})^{2/3}$, which we denote collectively by $\{\Phi_{II}\}$. Lastly, we will stabilize the rest of the Kähler moduli $\{\Phi_{III}\}$ by including the exponentially-suppressed last term.

Let $\Phi_I = \Phi_{I0}$ correspond to the minimum of

$$e^K \left(G^{a\bar{b}} D_a W \bar{D}_b \bar{W} + G^{\tau\bar{\tau}} D_\tau W \bar{D}_\tau \bar{W} \right). \tag{3.3}$$

Since this is positive definite, this means that $\{\Phi_{I0}\}$ is the solution to $D_a W = D_\tau W = 0$. We can evaluate the GVW superpotential and the first two terms of the Kähler potential at $\{\Phi_I\} = \{\Phi_{I0}\}$; write these values as W_0 and K_{cs}, respectively.

Next, let us include the $\mathcal{O}({(\mathcal{V})^{2/3}}^{-3})$ terms in the potential. Substituting $\Phi_I = \Phi_{I0} + \Phi_{I1}/(\mathcal{V})^{2/3}$, where $\{\Phi_{I1}\}$ can depend on $\{\Phi_{II}\}$, gives us

$$\begin{aligned} V &= \frac{e^{K_{cs}}}{{(\mathcal{V})^{2/3}}^4} \sum \mathcal{F}_1(\Phi_{I0})\, {\Phi_{I1}}^2 + V_{\alpha'} + V_{\mathrm{np1}} + V_{\mathrm{np2}} + V_{\mathrm{uplift}}, \\ &\approx V_{\alpha'} + V_{\mathrm{np1}} + V_{\mathrm{np2}} + V_{\mathrm{uplift}}, \end{aligned} \tag{3.4}$$

where now

$$\begin{aligned} V_{\alpha'} &\sim \frac{\xi}{{(\mathcal{V})^{2/3}}^3} e^{K_{cs}} |W_0|^2, \\ V_{\mathrm{np1}} &\sim \frac{(-D_{ssj} t^j) a_s^2 |A_s|^2 e^{-2a_s \tau_s} e^{K_{cs}}}{(\mathcal{V})^{2/3}}, \\ V'_{\mathrm{np2}} &= \frac{e^{K_{cs}}}{{(\mathcal{V})^{2/3}}^2} G^{\rho_s \bar{\rho}_l} i a_s (A_s e^{i a_s \rho_s} \bar{W}_0 \partial_{\bar{\rho}_l} K - \bar{A}_s e^{-i a_s \bar{\rho}_s} W_0 \partial_{\rho_l} \bar{K}). \end{aligned} \tag{3.5}$$

The minimum of the potential up to $\mathcal{O}({(\mathcal{V})^{2/3}}^{-3})$ is then given by

$$\begin{aligned} \Phi_{I\mathrm{min}} &= \Phi_{I0}, \\ \Phi_{II\mathrm{min}} &= \Phi_{II0}, \end{aligned} \tag{3.6}$$

with $\Phi_{II} = \Phi_{II0}$ the solution to $\partial_{\Phi_{II}} (V_{\alpha'} + V_{\mathrm{np1}} + V_{\mathrm{np2}} + V_{\mathrm{uplift}}) = 0$. Of course, we can continue this systematically order by order. Let the minimum value of the potential at the end of this second stage be V_0.

Now, let us include the exponentially suppressed part of the potential. Substituting[2]

$$\begin{aligned} \Phi_I &= \Phi_{I0} + \cdots + \frac{\Phi_{I2}}{{\mathcal{V}_0}^{2/3} e^{{(\mathcal{V})^{2/3}}_0}}, \\ \Phi_{II} &= \Phi_{II0} + \cdots + \Phi_{II2} \frac{{{(\mathcal{V})^{2/3}}_0}^{1/3}}{e^{{(\mathcal{V})^{2/3}}_0}}, \end{aligned} \tag{3.7}$$

[2]Since the volume is also a modulus we stabilized in the second stage, instead of using the full solution $(\mathcal{V})^{2/3}$, we use the leading term ${(\mathcal{V})^{2/3}}_0$ in our perturbation.

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where Φ_{I2} and Φ_{II2} can be dependent on $\{\Phi_{III}\}$ we get

$$\begin{aligned}
V &= V_0 + \frac{e^{K_{cs}}}{{(\mathcal{V})^{2/3}}_0^{4/3} e^{2{(\mathcal{V})^{2/3}}_0}} \sum \mathcal{F}_2(\Phi_{I0}, \Phi_{II0}) {\Phi_{I2}}^2 + \\
&+ \frac{e^{K_{cs}}}{{(\mathcal{V})^{2/3}}_0^{4/3} e^{2{(\mathcal{V})^{2/3}}_0}} \sum \mathcal{F}_3(\Phi_{I0}, \Phi_{II0}) {\Phi_{II2}}^2 + V_{\text{supp1}} + V_{\text{supp2}}, \\
&\approx V_0 + V_{\text{supp1}} + V_{\text{supp2}},
\end{aligned} \tag{3.8}$$

where V_{supp1} and V_{supp2} are independent of Φ_{I2} and Φ_{II2}. Thus, we have the minimum of the full potential at

$$\begin{aligned}
\Phi_{I\min} &= \Phi_{I0} + \cdots + \mathcal{O}\Big(\frac{1}{{(\mathcal{V})^{2/3}}_0^{2/3} e^{{(\mathcal{V})^{2/3}}_0}}\Big), \\
\Phi_{II\min} &= \Phi_{II0} + \cdots + \mathcal{O}\Big(\frac{{(\mathcal{V})^{2/3}}_0^{1/3}}{e^{\mathcal{V}_0}}\Big), \\
\Phi_{III\min} &= \Phi_{III0},
\end{aligned} \tag{3.9}$$

where Φ_{III0} is the solution to $\partial_{\Phi_{III}}(V_{\text{supp1}} + V_{\text{supp2}}) = 0$.

Neglecting volume suppressed terms, solution (3.9) can be written as

$$\Phi_{I\min} = \Phi_{I0}, \;\; \Phi_{II\min} = \Phi_{II0}, \;\; \Phi_{III\min} = \Phi_{III0}. \tag{3.10}$$

This means that solving $\partial_\alpha V = 0$ perturbatively can also be understood as an effective field theory approach: stabilize $\{\Phi_I\}$ by using only the leading term of the potential, integrate $\{\Phi_I\}$ out, stabilize $\{\Phi_{II}\}$ with the $\mathcal{O}({(\mathcal{V})^{2/3}}^{-3})$ terms, integrate them out, and lastly stabilize $\{\Phi_{III}\}$ by the remaining potential.

4. Toward modular inflation

Understanding the moduli stabilization problem using the language of effective theory, one can guess that the following will be valid approaches to simplify inflation analysis:

1. Integrating out the complex structure moduli and the axion-dilaton and then using the remaining theory to find inflation.

2. Integrating out the complex structure moduli, axion-dilaton, b_s, τ_s, and $(\mathcal{V})^{2/3}$ and then using the remaining theory to find inflation.

We will see that these approaches are valid by analyzing the equations of motion from the full theory. The necessary metric, inverse metric, and connection for the following subsections are given in Appendix B.

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4.1 First approach

Basically, we are trying to decouple complex structure moduli and the axion-dilaton from inflationary dynamics. First, we turn on *only* the fluxes needed to stabilize the complex structure moduli and the axion-dilaton, so that $\Phi_I = \Phi_{I0}$. Then, we incorporate the non-perturbative effects to create a potential for the Kähler moduli, which will be the inflatons.

Let $\Phi_I = \Phi_{I0} + \chi$, where $\Phi_{I0} \gg \chi$ and $\dot{\chi}(t=0) = 0$. Let us also assume that the inflatons are in the inflationary regime. Then, the fluctuations of the complex structure moduli about their minima satisfy the following equation at $t = 0$:

$$\ddot{\chi}^a + 2G^{ab}V_{,b} + 2G^{a\tau}V_{,\tau} = 0. \tag{4.1}$$

Since Φ_{I0} is at the minimum of the leading terms of the potential $\ddot{\chi}^a \sim \mathcal{O}((\mathcal{V})^{2/3^{-3}})$.

Similarly, for the axion-dilaton, at $t = 0$,

$$\ddot{\chi}^\tau + \Gamma^\tau{}_{\tau_l \tau_m}\dot{\phi}^l\dot{\phi}^m + \Gamma^\tau{}_{\tau_l \tau_s}\dot{\phi}^l\dot{\phi}^s + \Gamma^\tau{}_{\tau_s \tau_s}\dot{\phi}^s\dot{\phi}^s + 2\left(G^{\tau\bar{\tau}}V_{,\bar{\tau}} + G^{\tau a}V_{,a} + G^{\tau\tau_l}V_{,\tau_l} + G^{\tau\tau_s}V_{,\tau_s}\right) = 0, \tag{4.2}$$

where ϕ^i can be either the axion b_i or the 4-cycle modulus τ_i. Again, since Φ_{I0} is at the minimum of the leading terms of the potential, $G^{\tau\bar{\tau}}V_{,\bar{\tau}} \sim G^{\tau a}V_{,a} \sim \mathcal{O}((\mathcal{V})^{2/3^{-3}})$, while $G^{\tau\tau_l}V_{,\tau_l} \sim \mathcal{O}((\mathcal{V})^{2/3^{-13/3}})$ and $G^{\tau\tau_s}V_{,\tau_s} \sim \mathcal{O}((\mathcal{V})^{2/3^{-4}})$. Furthermore, since we are assuming that slow-roll obtains,

$$\begin{aligned}
\Gamma^\tau{}_{\tau_l \tau_m}\dot{\phi}^l\dot{\phi}^m &\lesssim \frac{1}{(\mathcal{V})^{2/3^{7/3}}}\frac{V}{G_{\tau_l\tau_m}} \sim \frac{1}{(\mathcal{V})^{2/3^4}}, \\
\Gamma^\tau{}_{\tau_l \tau_s}\dot{\phi}^l\dot{\phi}^s &\lesssim \frac{1}{(\mathcal{V})^{2/3^{8/3}}}\frac{V}{G_{\tau_l\tau_s}} \sim \frac{1}{(\mathcal{V})^{2/3^4}}, \\
\Gamma^\tau{}_{\tau_s \tau_s}\dot{\phi}^s\dot{\phi}^s &\lesssim \frac{1}{(\mathcal{V})^{2/3^2}}\frac{V}{G_{\tau_s\tau_s}} \sim \frac{1}{\mathcal{V}^4}.
\end{aligned} \tag{4.3}$$

Putting all these results together tells us that $\ddot{\chi}^\tau(t=0) \sim \mathcal{O}((\mathcal{V})^{2/3^{-3}})$.

At the next instant $\Delta t > 0$, $\dot{\chi}(\Delta t) = \dot{\chi}(0) + \ddot{\chi}\Delta t = \ddot{\chi}\Delta t$. This Taylor's expansion is valid only for small Δt, and since $H^{-1} \sim (\mathcal{V})^{2/3^{3/2}}$, the only small time scale in the theory is the string scale, which is equal to 1. Therefore, $\dot{\chi}^a \sim \dot{\chi}^\tau \sim \mathcal{O}((\mathcal{V})^{2/3^{-3}})$. The terms $3H\dot{\chi} \sim \mathcal{O}((\mathcal{V})^{2/3^{-9/2}})$, $\Gamma\,\dot{\chi}\dot{\chi} \sim \mathcal{O}((\mathcal{V})^{2/3^{-6}})$, and $\Gamma\,\dot{\chi}\dot{\phi} \sim \mathcal{O}((\mathcal{V})^{2/3^{-14/3}})$ still cannot compete with the derivative of the potential. Thus, as long as we are in the inflationary regime, $\ddot{\chi} \sim \mathcal{O}((\mathcal{V})^{2/3^{-3}})$ and $\dot{\chi} \sim \mathcal{O}(\mathcal{V}^{-3})$. Therefore, the contribution of the complex structure moduli and the axion-dilaton to the slow-roll parameter ϵ is

$$\epsilon_{cs} \sim \frac{G\,\dot{\chi}\dot{\chi}}{V} \sim \frac{1}{(\mathcal{V})^{2/3^3}}. \tag{4.4}$$

Thus, as long as $\frac{G\,\dot{\phi}\dot{\phi}}{V} < 1$, we do not have to worry about the contribution from $\{\Phi_I\}$. Therefore, we can decouple them from inflation analysis.

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Furthermore, calculating order of magnitudes, one should be able to see that the contributions of χ in the equation of motion of the inflatons are in fact negligible, thus validating equation (2.10).

An example of inflationary model where only the complex structure moduli and the axion-dilaton are stabilized can be found in [3][3].

4.2 Second approach

In this approach, after fixing $\{\Phi_I\}$, instead of turning on all the non-perturbative effects at once, we turn on only the one that corresponds to τ_s, so that $\Phi_I = \Phi_{I0} + \mathcal{O}(\frac{1}{(\mathcal{V})^{2/3}})$ and $\Phi_{II} = \Phi_{II0}$. Next, we turn on the rest of the non-perturbative effects to switch on the potential for $\{\Phi_{III}\}$. Let $\Phi_I = \Phi_{I0} + \chi_I$, where $\chi_I \ll \frac{1}{(\mathcal{V})^{2/3}}$ and $\dot{\chi}_I(t=0) = 0$, and let $\Phi_{II} = \Phi_{II0} + \chi_{II}$, where $\Phi_{II} \gg \chi_{II}$ and $\dot{\chi}_{II}(t=0) = 0$. Let us also assume that $\{\Phi_{III}\}$ is in the inflationary regime.

Note that the potential is exponentially suppressed. On the other hand, all geometry related quantities are $\sim (\mathcal{V})^{2/3^{\alpha}}$ and even if $\alpha > 0$, the geometrical quantities cannot compete with $e^{(\mathcal{V})^{2/3}}$ in the denominator. Because all we want to say is that the contribution from the kinetic energy of $\{\Phi_I\}$ and $\{\Phi_{II}\}$ terms toward ϵ is negligible, we can neglect the $(\mathcal{V})^{2/3^{\alpha}}$ factor and concentrate only on the exponential factor.

This allows us to infer that, $\ddot{\chi}_{I,II}(t=0) \sim e^{-(\mathcal{V})^{2/3}}$. Following the analysis done above for the first approach, we can say that a string time later $\dot{\chi}_{I,II} \sim e^{-(\mathcal{V})^{2/3}}$ so that $3H\dot{\chi} \sim e^{-2(\mathcal{V})^{2/3}}$ and $\Gamma\,\dot{\chi}\dot{\chi} \sim e^{-2(\mathcal{V})^{2/3}}$, which means that these terms are negligible compared to the other terms in the equation of motion. Thus, as long as $\{\Phi_{III}\}$ are in the inflationary regime, $\ddot{\chi} \sim e^{-(\mathcal{V})^{2/3}}$ and $\dot{\chi} \sim e^{-(\mathcal{V})^{2/3}}$. Therefore, the contribution of $\{\Phi_I\}$ and $\{\Phi_{II}\}$ to ϵ is

$$\epsilon_{\Phi_{I,II}} \sim \frac{G\,\dot{\chi}\dot{\chi}}{V} \sim \frac{1}{e^{(\mathcal{V})^{2/3}}}. \tag{4.5}$$

Therefore, we can decouple $\{\Phi_I\}$ and $\{\Phi_{II}\}$ from inflation analysis, and only worry about finding inflationary regime for $\{\Phi_{III}\}$.

An example where one can get a single-field inflation from this approach is given in [4].

5. Oscillation effects on the spectrum

We have seen that the large volume limit allows us to decouple a sufficient number of the moduli so that the problem of finding inflationary phases becomes tractable. However, when we say that the stabilized moduli are at the potential minimum, we mean that the *zero mode* is frozen. The fluctuations around this zero mode could be oscillating about the minimum and this may give rise to interesting effects [8]. In particular, these oscillations could imprint themselves on the CMB power spectrum; it should be noted that the exact nature of the effect depends on the model.

We need to be mindful of the requirement that the amplitude of oscillations be small enough that the energy density contained in them not disrupt the inflationary phase. This

[3] Even though [3] does not include the α' corrections, it should be possible to extend their analysis to include them.

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can be accomplished by just waiting long enough, since the energy density in these oscillations decays as that of non-relativistic matter.

What we would like to do is to calculate the power spectrum of the inflatons in the presence of these oscillations of the complex-structure moduli. To do this completely is a difficult problem, but we can at least estimate the order of magnitude of the effect within the large volume approximation scheme used above. We will see that the two approaches dealt with above can give rise to very different, and potentially measurable results.

5.1 First approach

Let us consider the quantum fluctuations of the inflatons, $\delta\phi^l$. A mode with wave number $\mathbf{k}$ has the following equation of motion

$$\begin{aligned}\delta\ddot{\phi}^l_{\mathbf{k}} &+ 3H\delta\dot{\phi}^l_{\mathbf{k}} + {\Gamma^{\tau_l}}_{tu}\left(\dot{\phi}^t\delta\dot{\phi}^u_{\mathbf{k}} + \delta\dot{\phi}^t_{\mathbf{k}}\dot{\phi}^u\right) + {\Gamma^{\tau_l}}_{tu,v}\dot{\phi}^t\dot{\phi}^u\delta\phi^v_{\mathbf{k}} + \left(G^{\tau_l t}V_{,tu} + {G^{\tau_l t}}_{,u}V_{,t}\right)\delta\phi^u_{\mathbf{k}} \\ &+ |k|^2 e^{-2Ht}\delta\phi^l_{\mathbf{k}} = 2\,{\Gamma^{\tau_l}}_{\tau t}\dot{\chi}^\tau\delta\dot{\phi}^t_{\mathbf{k}} + 2\,{\Gamma^{\tau_l}}_{\tau t,u}\dot{\chi}^\tau\dot{\phi}^t\delta\phi^u_{\mathbf{k}} \\ &+ \left({\Gamma^{\tau_l}}_{\tau\bar{\tau},t}\dot{\chi}^\tau\dot{\chi}^\tau + G^{\tau_l\tau}V_{,\tau t} + {G^{\tau_l\tau}}_{,t}V_{,\tau}\right)\delta\phi^t_{\mathbf{k}},\end{aligned} \tag{5.1}$$

where t, u, and v can be τ_s or τ_m with $m \neq s$. If there were no oscillating χ fields, the equation for the fluctuations would be the one above, but with the right hand side set to zero. Now let's turn to estimating the order of magnitude of the various terms in the Eq. (5.1):

$$\begin{aligned}\text{LHS} \sim\ & \delta\ddot{\phi}^l_{\mathbf{k}} + \left(\mathcal{O}(\frac{1}{{(\mathcal{V})^{2/3}}^{4/3}})\,\delta\dot{\phi}^s_{\mathbf{k}} + \mathcal{O}(\frac{1}{{(\mathcal{V})^{2/3}}^{7/3}})\delta\phi^s_{\mathbf{k}}\right) + \sum_{m\neq s}\left(\mathcal{O}(\frac{1}{{(\mathcal{V})^{2/3}}^{3/2}})\,\delta\dot{\phi}^m_{\mathbf{k}} + \mathcal{O}(\frac{1}{{(\mathcal{V})^{2/3}}^{3}})\,\delta\phi^m_{\mathbf{k}}\right) \\ &+ |k|^2 e^{-2Ht}\delta\phi^l_{\mathbf{k}}, \\ \text{RHS} \sim\ & \left(\mathcal{O}(\frac{1}{{(\mathcal{V})^{2/3}}^{13/3}})\,\delta\dot{\phi}^s_{\mathbf{k}} + \mathcal{O}(\frac{1}{\mathcal{V}^{11/3}})\delta\phi^s_{\mathbf{k}}\right) + \sum_{m\neq s}\left(\mathcal{O}(\frac{1}{{(\mathcal{V})^{2/3}}^{4}})\,\delta\dot{\phi}^m_{\mathbf{k}} + \mathcal{O}(\frac{1}{{(\mathcal{V})^{2/3}}^{13/3}})\,\delta\phi^m_{\mathbf{k}}\right).\end{aligned} \tag{5.2}$$

Similarly, for the mode $\delta\phi^s_{\mathbf{k}}$, we get

$$\begin{aligned}\delta\ddot{\phi}^s_{\mathbf{k}} &+ 3H\delta\dot{\phi}^s_{\mathbf{k}} + {\Gamma^{\tau_s}}_{tu}\left(\dot{\phi}^t\delta\dot{\phi}^u_{\mathbf{k}} + \delta\dot{\phi}^t_{\mathbf{k}}\dot{\phi}^u\right) + {\Gamma^{\tau_s}}_{tu,v}\dot{\phi}^t\dot{\phi}^u\delta\phi^v_{\mathbf{k}} + \left(G^{\tau_s t}V_{,tu} + {G^{\tau_s t}}_{,u}V_{,t}\right)\delta\phi^u_{\mathbf{k}} \\ &+ |k|^2 e^{-2Ht}\delta\phi^s_{\mathbf{k}} = 2\,{\Gamma^{\tau_s}}_{\tau t}\dot{\chi}^\tau\delta\dot{\phi}^t_{\mathbf{k}} + 2{\Gamma^{\tau_s}}_{\tau t,u}\dot{\chi}^\tau\dot{\phi}^t\delta\phi^u_{\mathbf{k}} \\ &+ \left({\Gamma^{\tau_s}}_{\tau\bar{\tau},t}\dot{\chi}^\tau\dot{\chi}^\tau + G^{\tau_s\tau}V_{,\tau t} + {G^{\tau_s\tau}}_{,t}V_{,\tau}\right)\delta\phi^t_{\mathbf{k}}.\end{aligned} \tag{5.3}$$

Examining the left and right hand sides of this equation gives us:

$$\begin{aligned}\text{LHS} \sim\ & \delta\ddot{\phi}^s_{\mathbf{k}} + \left(\mathcal{O}(\frac{1}{{(\mathcal{V})^{2/3}}^{3/2}})\,\delta\dot{\phi}^s_{\mathbf{k}} + \mathcal{O}(\frac{1}{{(\mathcal{V})^{2/3}}^{2}})\delta\phi^s_{\mathbf{k}}\right) + \sum_{m\neq s}\left(\mathcal{O}(\frac{1}{{(\mathcal{V})^{2/3}}^{5/3}})\,\delta\dot{\phi}^m_{\mathbf{k}} + \mathcal{O}(\frac{1}{{(\mathcal{V})^{2/3}}^{8/3}})\,\delta\phi^m_{\mathbf{k}}\right) \\ &+ |k|^2 e^{-2Ht}\delta\phi^s_{\mathbf{k}}, \\ \text{RHS} \sim\ & \mathcal{O}(\frac{1}{{(\mathcal{V})^{2/3}}^{4}})\left(\delta\dot{\phi}^s_{\mathbf{k}} + \delta\phi^s_{\mathbf{k}}\right) + \sum_{m\neq s}\mathcal{O}(\frac{1}{{(\mathcal{V})^{2/3}}^{14/3}})\left(\delta\dot{\phi}^m_{\mathbf{k}} + \delta\phi^m_{\mathbf{k}}\right).\end{aligned} \tag{5.4}$$

The equations of motion for the modes then become

$$\delta\ddot{\phi}^l_{\mathbf{k}} + |k|^2 e^{-2Ht}\delta\phi^l_{\mathbf{k}} + \left[\mathcal{O}(\frac{1}{{(\mathcal{V})^{2/3}}^{4/3}})\left(1 + \mathcal{O}(\frac{1}{{(\mathcal{V})^{2/3}}^{3}})\right)\delta\dot{\phi}^s_{\mathbf{k}} + \mathcal{O}(\frac{1}{{(\mathcal{V})^{2/3}}^{7/3}})\left(1 + \mathcal{O}(\frac{1}{\mathcal{V}^{4/3}})\right)\delta\phi^s_{\mathbf{k}}\right]$$

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$$+\sum_{m\neq s}\left[\mathcal{O}(\frac{1}{(\mathcal{V})^{2/3^{3/2}}})\left(1+\mathcal{O}(\frac{1}{(\mathcal{V})^{2/3^{5/2}}})\right)\delta\dot{\phi}^m_{\mathbf{k}}+\mathcal{O}(\frac{1}{(\mathcal{V})^{2/3^{3}}})\left(1+\mathcal{O}(\frac{1}{(\mathcal{V})^{2/3^{4/3}}})\right)\delta\phi^m_{\mathbf{k}}\right]=0, \quad (5.5)$$

$$\delta\ddot{\phi}^s_{\mathbf{k}}+|k|^2e^{-2Ht}\delta\phi^s_{\mathbf{k}}+\left[\mathcal{O}(\frac{1}{(\mathcal{V})^{2/3^{3/2}}})\left(1+\mathcal{O}(\frac{1}{(\mathcal{V})^{2/3^{5/2}}})\right)\delta\dot{\phi}^s_{\mathbf{k}}+\mathcal{O}(\frac{1}{(\mathcal{V})^{2/3^{2}}})\left(1+\mathcal{O}(\frac{1}{(\mathcal{V})^{2/3^{2}}})\right)\delta\phi^s_{\mathbf{k}}\right]$$

$$+\sum_{m\neq s}\left[\mathcal{O}(\frac{1}{(\mathcal{V})^{2/3^{5/3}}})\left(1+\mathcal{O}(\frac{1}{(\mathcal{V})^{2/3^{9/3}}})\right)\delta\dot{\phi}^m_{\mathbf{k}}+\mathcal{O}(\frac{1}{(\mathcal{V})^{2/3^{8/3}}})\left(1+\mathcal{O}(\frac{1}{(\mathcal{V})^{2/3^{2}}})\right)\delta\phi^m_{\mathbf{k}}\right]=0, \quad (5.6)$$

and we can neglect the contributions from χ. Therefore, in order to calculate the spectrum, we only need to solve

$$\text{LHS of eq. (5.1)} = \text{LHS of eq. (5.3)} = 0. \quad (5.7)$$

In this paper, we have assumed the use of non-perturbative effects from D3-instantons or gaugino condensations with low rank gauge group (i.e.: small N, a_i of $\mathcal{O}(1)$). However, there are many models where N needs to be large. For moderate N, as long as all a_i's are of the same order of magnitude, the hierarchy we have described still exist, only with smaller gaps between the stages. Therefore, most of our arguments here are applicable to the cases with larger N, with the exception that there is a possibility that the modification of the power spectrum in the second method can be larger and thus, observable. If N gets to a comparable size as the stabilized volume, then not only our arguments are no longer valid, but terms from higher order instantons will also no longer be suppressed.

Furthermore, we can estimate the effect of the oscillation of $\{\chi\}$ in the spectrum. Since all the coefficients in front of $\delta\phi_{\mathbf{k}}$ and $\delta\dot{\phi}_{\mathbf{k}}$ are in the form of $A(1+B)$ with $B \ll 1$, the ratio of the effect of $\{\chi\}$ in the spectrum with the spectrum will be the biggest B. Thus,

$$\frac{\delta P}{P}(|\mathbf{k}|) \sim \frac{1}{\mathcal{V}^{4/3}}. \quad (5.8)$$

For the model in [3], the volume is $(\mathcal{V})^{2/3} = 99$ in string units. Thus, the change in the spectrum from complex-structure moduli and the axion-dilaton is of order 10^{-2}. Since the current experiment can measure $\delta P/P$ up to order $10^{-3} - 10^{-2}$, it is necessary to calculate this effect in model [3][4].

5.2 Second approach

For the second approach, since $\dot{\chi} \sim e^{-(\mathcal{V})^{2/3}}$ and $V \sim e^{-(\mathcal{V})^{2/3}}$ then in the equation of motion, the contribution of χ will also be of order $e^{-(\mathcal{V})^{2/3}}$. On the other hand, the other terms are of order $\sqrt{V} \sim e^{-\mathcal{V}/2}$. Thus, the effect of χ's oscillation on the spectrum is

$$\frac{\delta P}{P}(|\mathbf{k}|) \sim \frac{1}{e^{\mathcal{V}/2}}. \quad (5.9)$$

Since the volume is at least $10^2 - 10^3$ string units, this effect is too small to be measured.

[4]To do so, one has to extend the analysis in [3] to include the α' corrections.

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6. Discussion

Motivated by the scale hierarchy of the moduli in the large volume scheme, we have approached the problem of moduli stabilization by dividing it into several stages. We would like to emphasize that the decoupling in the moduli stabilization procedure does *not* come from any underlying assumptions such as suggested in the original KKLT procedure [15]. The decoupling comes from approaching this problem perturbatively using $1/(\mathcal{V})^{2/3}$-expansion.

We also have shown that the fields that are stabilized in the earlier stage(s) can be integrated out of the theory, thus reducing the number of possible inflatons and rendering the search for an inflationary phase in this theory easier [4].

While we did not pursue the detailed analysis of this possibility, we also have seen that the oscillation of the stabilized fields could, at least in principle, modify the scalar power spectrum. For the second method, this modification is small and cannot be measured by our current experiments. Thus, we can calculate the power spectrum as if there is no oscillating fields in the background. However, we saw that in the first approach, there is the possibility that an effect could be observable. This merits further study.

As noted in [6], our arguments may not be completely airtight. The treatment of the the loop determinant A_i as a constant, may not be warranted. In particular, if A_i depends on the Kähler moduli, our argument might not be valid. Since polynomial dependence on the Kähler moduli is unlikely, we only have to worry for the case $A_s \sim \mathcal{V}^{\alpha}$ (we do not have to worry for A_l due to the exponential-suppression on the denominator). In that case, we can save our argument by redefining $\tau_s \sim (\alpha+1)\ln\mathcal{V}$.

As noted in [6], our arguments may not be completely airtight. The treatment of the the loop determinant A_i as a constant, may not be warranted. In particular, if A_i depends on the Kähler moduli, our argument might not be valid. If $A_s \sim \mathcal{V}^{\alpha}$ (we do not have to worry for A_l due to the exponential-suppression on the denominator), we can save our argument by redefining $\tau_s \sim (\alpha+1)\ln\mathcal{V}$. However, the polynomial dependence on the Kähler moduli is unlikely due to holomorphy and shift symmetry.

From the point of view of inflationary dynamics, there is also an issue of the likelihood of the initial conditions. Given that $\{\Phi_I\}$ for the first approach (or $\{\Phi_I\}$ and $\{\Phi_{II}\}$ for the second approach) are at the minimum, how likely will it be for the rest of the moduli to be in the slow-roll regime? This requires further analysis.

We also would like to emphasize that our approaches might not be the only way to simplify the analysis of inflation in flux compactifications. A different approach would be to change the definition of the large volume limit. Nevertheless, the trick will be the same, namely exploitation of the scale hierarchy of the moduli. For example, by defining large volume limit as the limit where only one $\tau_l \to \infty$ and the rest $\tau_i \sim \ln\mathcal{V}$, we can get a single-field inflation like in [4] without having to restrict ourselves to Calabi-Yau orientifolds with $h^{1,1} = 2$. Thus, 'decoupling' a field ψ from the inflationary dynamics by 'constraining' it to the minima, while letting inflaton ϕ rolls over a potential $V(\phi)$ that has a comparable scale to the potential for ψ will not be valid.

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In the literature, there are inflationary models where only axions are integrated out (e.g.: [21, 18]). This is done under the assumption that the axions are heavier than the rest of the Kähler moduli. However, as we have discussed in the previous paragraph, this assumption might be of suspect. To validate these models, it is crucial to find an approach where there is a hierarchy between the potential of the axions and the potential for the rest of the Kähler moduli.

It would be interesting to see whether there is a correlation between the number of left-over moduli and the power spectrum. If there is, then as cosmological data becomes more precise, it would not be surprising that one can put constraints on the extra dimensions using cosmological data (an initial attempt at falsifying stringy inflationary models was given in ref. [18]).

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A. Slow-roll condition for multi-field inflation

Consider an FRW background. From Einstein equations, we can get the evolution of the scale factor

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{6}, \tag{A.1}$$

where assuming homogeneity and isotropy, the energy density of the system $\rho = G_{AB}\dot{\phi}^A\dot{\phi}^B$ $+V$. Using Friedmann equation (A.1) and the mass conservation, we get the equation for the acceleration of the scale factor

$$\frac{\ddot{a}}{a} = -\frac{\rho + 3p}{6}, \tag{A.2}$$

where $p = G_{AB}\dot{\phi}^A\dot{\phi}^B - V$.

Inflation is defined as an epoch where $\ddot{a}/a > 0$. Since

$$\frac{\ddot{a}}{a} = H^2(1-\epsilon); \;\; \epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{G_{AB}\dot{\phi}^A\dot{\phi}^B}{H^2}, \tag{A.3}$$

inflation $\Leftrightarrow \epsilon < 1$. Notice that from equation (A.2), inflation also means that

$$2G_{AB}\dot{\phi}^A\dot{\phi}^B < V. \tag{A.4}$$

If we further assume that

$$2G^{AB}V_{,B} \gg \ddot{\Phi}^A + \Gamma^A{}_{BC}\dot{\Phi}^B\dot{\Phi}^C, \tag{A.5}$$

we get

$$\epsilon = \frac{G^{AB}V_{,A}V_{,B}}{4V^2}. \tag{A.6}$$

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Up until this point, this analysis resembles the one for the case of single-field inflation. In single-field inflation, one will get another condition $\eta < 1$ for inflation by demanding equation (A.4) is consistent with equation (A.5). However, in multi-field inflation, one is not able to define η in that manner. Since our main discussion does not involve η, we will not discuss this matter any further[5].

B. The dependence of metric, inverse metric, and connection on classical volume

In the large volume limit, the Kähler potential becomes

$$\begin{aligned} K &= 3\phi_0 - \log\left[-i\int_M \Omega\wedge\bar{\Omega}\right] - \log\left[-i(\tau-\bar{\tau})\right] - 2\log\left[1+\frac{e^{3\phi_0/2}}{(\mathcal{V})^{2/3}}\frac{\xi}{2}\left(\frac{-i(\tau-\bar{\tau})}{2}\right)^{3/2}\right] \\ &\quad -2\log(\mathcal{V})^{2/3}, \\ &= 3\phi_0 - \log\left[-i\int_M \Omega\wedge\bar{\Omega}\right] - \log\left[-i(\tau-\bar{\tau})\right] - \frac{2e^{3\phi_0/2}}{(\mathcal{V})^{2/3}}\frac{\xi}{2}\left(\frac{-i(\tau-\bar{\tau})}{2}\right)^{3/2} \\ &\quad -2\log(\mathcal{V})^{2/3}. \end{aligned} \tag{B.1}$$

Noticing that the relation between volume and the four-cycle moduli is like $(\mathcal{V})^{2/3} \sim \tau_i t^i$, and that $(\mathcal{V})^{2/3} \sim {\tau_l}^{3/2}$, $l \neq s$ in the large volume limit, we get

$$\frac{\partial(\mathcal{V})^{2/3}}{\partial\tau_l} \sim t^l \sim {\tau_l}^{1/2} \sim {(\mathcal{V})^{2/3}}^{1/3}, \tag{B.2}$$

for $l \neq s$, and

$$\frac{\partial(\mathcal{V})^{2/3}}{\partial\tau_s} \sim t^s \sim {\tau_s}^{1/2} \sim \mathcal{O}(1). \tag{B.3}$$

Therefore, the components of the metric become

$$\begin{aligned} &G_{\tau\bar{\tau}} = \mathcal{O}(1), \quad G_{\tau\bar{\rho_l}} \sim \frac{1}{{(\mathcal{V})^{2/3}}^{5/3}}, \quad G_{\tau\bar{\rho_s}} \sim \frac{1}{{(\mathcal{V})^{2/3}}^{2}}, \\ &G_{\rho_l\bar{\rho_m}} \sim \frac{1}{{(\mathcal{V})^{2/3}}^{4/3}}, \quad G_{\rho_l\bar{\rho_s}} \sim \frac{1}{{(\mathcal{V})^{2/3}}^{5/3}}, \quad G_{\rho_s\bar{\rho_s}} \sim \frac{1}{(\mathcal{V})^{2/3}}. \end{aligned} \tag{B.4}$$

The components of the inverse metric are given in [20].

$$\begin{aligned} &G^{\tau\bar{\tau}} = \mathcal{O}(1), \quad G^{\tau\bar{\rho_l}} \sim \frac{1}{{(\mathcal{V})^{2/3}}^{2/3}}, \quad G^{\tau\bar{\rho_s}} \sim \frac{1}{(\mathcal{V})^{2/3}}, \\ &G^{\rho_l\bar{\rho_m}} \sim {(\mathcal{V})^{2/3}}^{4/3}, \quad G^{\rho_l\bar{\rho_s}} \sim {(\mathcal{V})^{2/3}}^{2/3}, \quad G^{\rho_s\bar{\rho_s}} \sim (\mathcal{V})^{2/3}. \end{aligned} \tag{B.5}$$

Let us remind ourselves that we need to change variables from the complex moduli fields to the real scalar fields for calculation in Section 4. Since the components of the metric (and inverse metric) for the real scalar fields are of the same order with the corresponding

[5]One possibility in defining η is given in [19].

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components for the complex moduli fields, we will adopt a somewhat loose notation for the connection. The components of the connection necessary for calculation in Section 4 and Section 5 are

$$\begin{aligned}
\Gamma^{\tau}{}_{\tau_l\tau_m} &= \frac{1}{2}\,G^{\tau\bar{\tau}}\,(\,G_{\tau_l\bar{\tau},\tau_m} + G_{\tau_m\bar{\tau},\tau_l} - G_{\tau_l\tau_m,\bar{\tau}}\,)\\
&\quad+\frac{1}{2}\,G^{\tau\tau_n}\,(\,G_{\tau_l\tau_n,\tau_m} + G_{\tau_m\tau_n,\tau_l} - G_{\tau_l\tau_m,\tau_n}\,)\\
&\quad+\frac{1}{2}\,G^{\tau\tau_s}\,(\,G_{\tau_l\tau_s,\tau_m} + G_{\tau_m\tau_s,\tau_l} - G_{\tau_l\tau_m,\tau_s}\,)\,,\\
&\sim \mathcal{O}(1)\,\frac{1}{(\mathcal{V})^{2/3^{7/3}}} + \frac{1}{\mathcal{V}^{2/3}}\,\frac{1}{(\mathcal{V})^{2/3^{2}}} + \frac{1}{(\mathcal{V})^{2/3}}\,\frac{1}{(\mathcal{V})^{2/3^{7/3}}},\\
&\sim \frac{1}{(\mathcal{V})^{2/3^{7/3}}}, \qquad\qquad \text{(B.6)}\\
\Gamma^{\tau}{}_{\tau_l\tau_s} &= \frac{1}{2}\,G^{\tau\bar{\tau}}\,(\,G_{\tau_l\bar{\tau},\tau_s} + G_{\tau_s\bar{\tau},\tau_l} - G_{\tau_l\tau_s,\bar{\tau}}\,)\\
&\quad+\frac{1}{2}\,G^{\tau\tau_m}\,(\,G_{\tau_l\tau_m,\tau_s} + G_{\tau_s\tau_m,\tau_l} - G_{\tau_l\tau_s,\tau_m}\,) + \frac{1}{2}\,G^{\tau\tau_s}\,G_{\tau_s\tau_s,\tau_l},\\
&\sim \mathcal{O}(1)\,\frac{1}{(\mathcal{V})^{2/3^{8/3}}} + \frac{1}{\mathcal{V}^{2/3}}\,\frac{1}{(\mathcal{V})^{2/3^{7/3}}} + \frac{1}{(\mathcal{V})^{2/3}}\,\frac{1}{(\mathcal{V})^{2/3^{5/3}}},\\
&\sim \frac{1}{(\mathcal{V})^{2/3^{8/3}}}, \qquad\qquad \text{(B.7)}\\
\Gamma^{\tau}{}_{\tau_s\tau_s} &= \frac{1}{2}\,G^{\tau\bar{\tau}}\,(\,2G_{\tau_s\bar{\tau},\tau_s} - G_{\tau_s\tau_s,\bar{\tau}}\,)\\
&\quad+\frac{1}{2}\,G^{\tau\tau_l}\,(\,2G_{\tau_s\tau_l,\tau_s} - G_{\tau_s\tau_s,\tau_l}\,) + \frac{1}{2}\,G^{\tau\tau_s}\,G_{\tau_s\tau_s,\tau_s},\\
&\sim \mathcal{O}(1)\,\frac{1}{(\mathcal{V})^{2/3^{2}}} + \frac{1}{\mathcal{V}^{2/3}}\,\frac{1}{(\mathcal{V})^{2/3^{5/3}}} + \frac{1}{(\mathcal{V})^{2/3}}\,\frac{1}{(\mathcal{V})^{2/3}},\\
&\sim \frac{1}{(\mathcal{V})^{2/3^{2}}}, \qquad\qquad \text{(B.8)}\\
\Gamma^{\tau_s}{}_{\tau_l\tau_m} &= \frac{1}{2}\,G^{\tau_s\bar{\tau}}\,(\,G_{\tau_l\bar{\tau},\tau_m} + G_{\tau_m\bar{\tau},\tau_l} - G_{\tau_l\tau_m,\bar{\tau}}\,)\\
&\quad+\frac{1}{2}\,G^{\tau_s\tau_n}\,(\,G_{\tau_l\tau_n,\tau_m} + G_{\tau_m\tau_n,\tau_l} - G_{\tau_l\tau_m,\tau_n}\,)\\
&\quad+\frac{1}{2}\,G^{\tau_s\tau_s}\,(\,G_{\tau_l\tau_s,\tau_m} + G_{\tau_m\tau_s,\tau_l} - G_{\tau_l\tau_m,\tau_s}\,)\,,\\
&\sim \frac{1}{(\mathcal{V})^{2/3}}\,\frac{1}{(\mathcal{V})^{2/3^{7/3}}} + (\mathcal{V})^{2/3^{2/3}}\,\frac{1}{(\mathcal{V})^{2/3^{2}}} + (\mathcal{V})^{2/3}\,\frac{1}{(\mathcal{V})^{2/3^{7/3}}},\\
&\sim \frac{1}{(\mathcal{V})^{2/3^{4/3}}}, \qquad\qquad \text{(B.9)}\\
\Gamma^{\tau_s}{}_{\tau_l\tau_s} &= \frac{1}{2}\,G^{\tau_s\bar{\tau}}\,(\,G_{\tau_l\bar{\tau},\tau_s} + G_{\tau_s\bar{\tau},\tau_l} - G_{\tau_l\tau_s,\bar{\tau}}\,)\\
&\quad+\frac{1}{2}\,G^{\tau_s\tau_m}\,(\,G_{\tau_l\tau_m,\tau_s} + G_{\tau_s\tau_m,\tau_l} - G_{\tau_l\tau_s,\tau_m}\,) + \frac{1}{2}\,G^{\tau_s\tau_s}\,G_{\tau_s\tau_s,\tau_l},
\end{aligned}$$

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$$
\begin{aligned}
&\sim \frac{1}{(\mathcal{V})^{2/3}}\,\frac{1}{(\mathcal{V})^{2/3^{8/3}}} + (\mathcal{V})^{2/3^{2/3}}\,\frac{1}{(\mathcal{V})^{2/3^{7/3}}} + (\mathcal{V})^{2/3}\,\frac{1}{(\mathcal{V})^{2/3^{5/3}}},\\
&\sim \frac{1}{(\mathcal{V})^{2/3^{2/3}}}, \qquad \text{(B.10)}\\
\Gamma^{\tau_s}{}_{\tau_s\tau_s} &= \frac{1}{2}\,G^{\tau_s\bar{\tau}}\,(\,2G_{\tau_s\bar{\tau},\tau_s} - G_{\tau_s\tau_s,\bar{\tau}}\,)\\
&\quad+\frac{1}{2}\,G^{\tau_s\tau_l}\,(\,2G_{\tau_s\tau_l,\tau_s} - G_{\tau_s\tau_s,\tau_l}\,) + \frac{1}{2}\,G^{\tau_s\tau_s}\,G_{\tau_s\tau_s,\tau_s},\\
&\sim \frac{1}{(\mathcal{V})^{2/3}}\,\frac{1}{(\mathcal{V})^{2/3^{2}}} + (\mathcal{V})^{2/3^{2/3}}\,\frac{1}{(\mathcal{V})^{2/3^{5/3}}} + (\mathcal{V})^{2/3}\,\frac{1}{(\mathcal{V})^{2/3}},\\
&\sim \frac{1}{(\mathcal{V})^{2/3^{2}}}, \qquad \text{(B.11)}\\
\Gamma^{\tau_s}{}_{\tau\tau_l} &= \frac{1}{2}\,G^{\tau_s\bar{\tau}}\,(\,G_{\tau\bar{\tau},\tau_l} + G_{\tau_l\bar{\tau},\tau} - G_{\tau\tau_l,\bar{\tau}}\,)\\
&\quad+\frac{1}{2}\,G^{\tau_s\tau_m}\,(\,G_{\tau\tau_m,\tau_l} + G_{\tau_l\tau_m,\tau} - G_{\tau\tau_l,\tau_m}\,)\\
&\quad+\frac{1}{2}\,G^{\tau_s\tau_s}\,(\,G_{\tau\tau_s,\tau_l} + G_{\tau_l\tau_s,\tau} - G_{\tau\tau_l,\tau_s}\,),\\
&\sim \frac{1}{(\mathcal{V})^{2/3}}\,\frac{1}{(\mathcal{V})^{2/3^{5/3}}} + (\mathcal{V})^{2/3^{2/3}}\,\frac{1}{(\mathcal{V})^{2/3^{7/3}}} + (\mathcal{V})^{2/3}\,\frac{1}{(\mathcal{V})^{2/3^{8/3}}},\\
&\sim \frac{1}{(\mathcal{V})^{2/3^{5/3}}}, \qquad \text{(B.12)}\\
\Gamma^{\tau_s}{}_{\tau\tau_s} &= \frac{1}{2}\,G^{\tau_s\bar{\tau}}\,(\,G_{\tau\bar{\tau},\tau_s} + G_{\tau_s\bar{\tau},\tau} - G_{\tau\tau_s,\bar{\tau}}\,)\\
&\quad+\frac{1}{2}\,G^{\tau_s\tau_l}\,(\,G_{\tau\tau_l,\tau_s} + G_{\tau_s\tau_l,\tau} - G_{\tau\tau_s,\tau_l}\,) + \frac{1}{2}\,G^{\tau_s\tau_s}\,G_{\tau_s\tau_s,\tau},\\
&\sim \frac{1}{(\mathcal{V})^{2/3}}\,\frac{1}{(\mathcal{V})^{2/3^{2}}} + (\mathcal{V})^{2/3^{2/3}}\,\frac{1}{(\mathcal{V})^{2/3^{8/3}}} + (\mathcal{V})^{2/3}\,\frac{1}{(\mathcal{V})^{2/3^{2}}},\\
&\sim \frac{1}{(\mathcal{V})^{2/3}}, \qquad \text{(B.13)}\\
\Gamma^{\tau_s}{}_{\tau\bar{\tau}} &= \frac{1}{2}\,G^{\tau_s\bar{\tau}}\,G_{\tau\bar{\tau},\bar{\tau}} + \frac{1}{2}\,G^{\tau_s\tau_l}\,(\,G_{\tau\tau_l,\bar{\tau}} + G_{\bar{\tau}\tau_l,\tau} - G_{\tau\bar{\tau},\tau_l}\,)\\
&\quad+\frac{1}{2}\,G^{\tau_s\tau_s}\,(\,G_{\tau\tau_s,\bar{\tau}} + G_{\bar{\tau}\tau_s,\tau} - G_{\tau\bar{\tau},\tau_s}\,),\\
&\sim \frac{1}{(\mathcal{V})^{2/3}}\,\mathcal{O}(1) + (\mathcal{V})^{2/3^{2/3}}\,\frac{1}{(\mathcal{V})^{2/3^{5/3}}} + (\mathcal{V})^{2/3}\,\frac{1}{(\mathcal{V})^{2/3^{2}}},\\
&\sim \frac{1}{(\mathcal{V})^{2/3}}, \qquad \text{(B.14)}\\
\Gamma^{\tau_l}{}_{\tau_m\tau_n} &= \frac{1}{2}\,G^{\tau_l\bar{\tau}}\,(\,G_{\tau_m\bar{\tau},\tau_n} + G_{\tau_n\bar{\tau},\tau_m} - G_{\tau_m\tau_n,\bar{\tau}}\,)\\
&\quad+\frac{1}{2}\,G^{\tau_l\tau_o}\,(\,G_{\tau_m\tau_o,\tau_n} + G_{\tau_n\tau_o,\tau_m} - G_{\tau_m\tau_n,\tau_o}\,)\\
&\quad+\frac{1}{2}\,G^{\tau_l\tau_s}\,(\,G_{\tau_m\tau_s,\tau_n} + G_{\tau_n\tau_s,\tau_m} - G_{\tau_m\tau_n,\tau_s}\,),
\end{aligned}
$$

$$
\begin{aligned}
&\sim \frac{1}{(\mathcal{V})^{2/3^{2/3}}}\,\frac{1}{(\mathcal{V})^{2/3^{7/3}}} + \mathcal{V}^{4/3}\,\frac{1}{(\mathcal{V})^{2/3^{2}}} + (\mathcal{V})^{2/3^{2/3}}\,\frac{1}{(\mathcal{V})^{2/3^{7/3}}},\\
&\sim \frac{1}{(\mathcal{V})^{2/3^{2/3}}},
\end{aligned}
\tag{B.15}
$$

$$
\begin{aligned}
\Gamma^{\tau_l}{}_{\tau_m\tau_s} &= \frac{1}{2}\,G^{\tau_l\bar{\tau}}\,(\,G_{\tau_m\bar{\tau},\tau_s} + G_{\tau_s\bar{\tau},\tau_m} - G_{\tau_m\tau_s,\bar{\tau}}\,)\\
&\quad + \frac{1}{2}\,G^{\tau_l\tau_n}\,(\,G_{\tau_m\tau_n,\tau_s} + G_{\tau_s\tau_n,\tau_m} - G_{\tau_m\tau_s,\tau_n}\,) + \frac{1}{2}\,G^{\tau_l\tau_s}G_{\tau_s\tau_s,\tau_m},\\
&\sim \frac{1}{(\mathcal{V})^{2/3^{2/3}}}\,\frac{1}{(\mathcal{V})^{2/3^{8/3}}} + \mathcal{V}^{4/3}\,\frac{1}{(\mathcal{V})^{2/3^{7/3}}} + (\mathcal{V})^{2/3^{2/3}}\,\frac{1}{(\mathcal{V})^{2/3^{5/3}}},\\
&\sim \frac{1}{(\mathcal{V})^{2/3}},
\end{aligned}
\tag{B.16}
$$

$$
\begin{aligned}
\Gamma^{\tau_l}{}_{\tau_s\tau_s} &= \frac{1}{2}\,G^{\tau_l\bar{\tau}}\,(\,2G_{\tau_s\bar{\tau},\tau_s} - G_{\tau_s\tau_s,\bar{\tau}}\,)\\
&\quad + \frac{1}{2}\,G^{\tau_l\tau_m}\,(\,2G_{\tau_s\tau_m,\tau_s} - G_{\tau_s\tau_s,\tau_m}\,) + \frac{1}{2}\,G^{\tau_l\tau_s}\,G_{\tau_s\tau_s,\tau_s},\\
&\sim \frac{1}{(\mathcal{V})^{2/3^{2/3}}}\,\frac{1}{(\mathcal{V})^{2/3^{2}}} + \mathcal{V}^{4/3}\,\frac{1}{(\mathcal{V})^{2/3^{5/3}}} + (\mathcal{V})^{2/3^{2/3}}\,\frac{1}{(\mathcal{V})^{2/3}},\\
&\sim \frac{1}{(\mathcal{V})^{2/3^{1/3}}},
\end{aligned}
\tag{B.17}
$$

$$
\begin{aligned}
\Gamma^{\tau_l}{}_{\tau\tau_m} &= \frac{1}{2}\,G^{\tau_l\bar{\tau}}\,(\,G_{\tau\bar{\tau},\tau_m} + G_{\tau_m\bar{\tau},\tau} - G_{\tau\tau_m,\bar{\tau}}\,)\\
&\quad + \frac{1}{2}\,G^{\tau_l\tau_n}\,(\,G_{\tau\tau_n,\tau_m} + G_{\tau_m\tau_n,\tau} - G_{\tau\tau_m,\tau_n}\,)\\
&\quad + \frac{1}{2}\,G^{\tau_l\tau_s}\,(\,G_{\tau\tau_s,\tau_m} + G_{\tau_m\tau_s,\tau} - G_{\tau\tau_m,\tau_s}\,),\\
&\sim \frac{1}{(\mathcal{V})^{2/3^{2/3}}}\,\frac{1}{(\mathcal{V})^{2/3^{5/3}}} + \mathcal{V}^{4/3}\,\frac{1}{(\mathcal{V})^{2/3^{7/3}}} + (\mathcal{V})^{2/3^{2/3}}\,\frac{1}{(\mathcal{V})^{2/3^{8/3}}},\\
&\sim \frac{1}{(\mathcal{V})^{2/3}},
\end{aligned}
\tag{B.18}
$$

$$
\begin{aligned}
\Gamma^{\tau_l}{}_{\tau\tau_s} &= \frac{1}{2}\,G^{\tau_l\bar{\tau}}\,(\,G_{\tau\bar{\tau},\tau_s} + G_{\tau_s\bar{\tau},\tau} - G_{\tau\tau_s,\bar{\tau}}\,)\\
&\quad + \frac{1}{2}\,G^{\tau_l\tau_m}\,(\,G_{\tau\tau_m,\tau_s} + G_{\tau_s\tau_m,\tau} - G_{\tau\tau_s,\tau_m}\,) + \frac{1}{2}\,G^{\tau_l\tau_s}\,G_{\tau_s\tau_s,\tau}\\
&\sim \frac{1}{(\mathcal{V})^{2/3^{2/3}}}\,\frac{1}{(\mathcal{V})^{2/3^{2}}} + \mathcal{V}^{4/3}\,\frac{1}{(\mathcal{V})^{2/3^{8/3}}} + (\mathcal{V})^{2/3^{2/3}}\,\frac{1}{(\mathcal{V})^{2/3^{2}}},\\
&\sim \frac{1}{(\mathcal{V})^{2/3^{4/3}}},
\end{aligned}
\tag{B.19}
$$

$$
\begin{aligned}
\Gamma^{\tau_l}{}_{\tau\bar{\tau}} &= \frac{1}{2}\,G^{\tau_l\bar{\tau}}\,G_{\tau\bar{\tau},\bar{\tau}} + \frac{1}{2}\,G^{\tau_l\tau_m}\,(\,G_{\tau\tau_m,\bar{\tau}} + G_{\bar{\tau}\tau_m,\tau} - G_{\tau\bar{\tau},\tau_m}\,)\\
&\quad + \frac{1}{2}\,G^{\tau_l\tau_s}\,(\,G_{\tau\tau_s,\bar{\tau}} + G_{\bar{\tau}\tau_s,\tau} - G_{\tau\bar{\tau},\tau_s}\,),
\end{aligned}
$$

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$$\sim \frac{1}{(\mathcal{V})^{2/3^{2/3}}}\,\mathcal{O}(1) + (\mathcal{V})^{2/3^{4/3}}\,\frac{1}{(\mathcal{V})^{2/3^{5/3}}} + (\mathcal{V})^{2/3^{2/3}}\,\frac{1}{(\mathcal{V})^{2/3^{2}}},$$
$$\sim \frac{1}{(\mathcal{V})^{2/3^{1/3}}}. \tag{B.20}$$

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